

Anomalous chromomagnetic moment of quarks

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Abstract

We do a perturbative calculation of the anomalous chromomagnetic dipole moment of quarks at one-loop, also considering the effect of a small gauge-invariant mass of the gluon. We find partial agreement with a previous calculation, as well as a divergence. We explain these results by noting that perturbation theory is not valid at the energy scales where these calculations were done, and proceed to give the results at the M_Z scale. We find significant variation, of the anomalous moment of the light quarks, as a function of gluon mass.

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I. INTRODUCTION

The Linear Hadron Collider (LHC), with its recent discovery of a 125 GeV Higgs boson [1, 2], has completed the observation of all fundamental particles of the Standard Model. But many questions remain unanswered about the properties of the low energy particle universe. The origin of neutrino mass and mechanism of family symmetry breaking are the ones that get the most attention, along with that of whether there are more particles to be found as the energy and luminosity of the LHC increases. We are interested in another question, one that concerns quantum chromodynamics (QCD). The LHC is a QCD machine, and it is likely to provide a unique window to precision QCD.

The anomalous magnetic dipole moments of the electron and the muon are among the most precisely computed and measured quantities in quantum electrodynamics. The corresponding quantity for QCD, the anomalous chromomagnetic dipole moment (CMDM) of quarks, is not so precisely known. Starting from a general effective Lagrangian containing the anomalous couplings [3, 4], one may look for the contribution of new physics in various processes by analyzing available data. While some bounds have been obtained this way for the new physics contribution to the CMDM of the top quark [5–16], this particular quantity has received only sparse attention from the community so far. Indeed, it was only very recently that an experimental collaboration did an analysis of the top-quark CMDM for the first time [17]. Even less attention seems to have been paid to the anomalous CMDM of the other quarks, although some calculations have been done for light quarks in the context of computing the contribution of quark CMDM to nucleon anomalous magnetic moments and electromagnetic form factors [18]. As the LHC starts measuring QCD quantities more accurately, we can expect more interest in anomalous chromomagnetic dipole moments, and more generally form factors, of quarks.

Another quantity that is likely to be known more precisely through measurements at the LHC is the mass of the gluon. There is no Higgs mechanism for QCD as the color symmetry of the theory is unbroken [19]. On the other hand, a Proca mass term for the gluon breaks gauge symmetry, leading to a breakdown of renormalizability as well as violation of unitarity at high energy by certain tree level amplitudes. However, there are some ways a gluon can be massive in a gauge-invariant manner.

One is due to Cornwall [20], who suggested that non-zero gluon mass can be generated

dynamically in a theory in which the color symmetry remains unbroken. A dynamically generated gluon mass depends on momentum; it must also vanish at large momentum so as to maintain renormalizability of the theory. Another model of a massive gluon is the Curci-Ferrari model [22], which has a Proca mass term as well as a ‘gauge-fixing’ term, uses a quartic ghost interaction to make it renormalizable (but not unitary) [23]. Another is the topological mass generation mechanism [24], in which an antisymmetric tensor provides a mass to the gauge boson via a derivative coupling without breaking global or gauge symmetries. This model also appears to be unitary and renormalizable [25–28].

Early analyses led to estimates of the gluon mass over a large range, from 500 ± 200 MeV [20] using numerical calculations of the mass gap, to $\simeq 800$ MeV [29] on the basis of strong suppression of the end point of the photon spectrum in radiative J/ψ decays, to $\simeq 1$ GeV [30] based on analysis of photon spectra in the processes $J/\psi \rightarrow \gamma X$ and $\Upsilon \rightarrow \gamma X$. More recently, an analysis of data from free quark searches found a nominal upper limit of $\mathcal{O}(1)$ MeV [32]. It is clear from these studies that the question of whether the gluon has a mass is yet to be settled experimentally. We can expect that precision measurements of QCD quantities, such as the anomalous CMDM, will lead to setting bounds on gluon mass.

In this paper we calculate the anomalous CMDM of quarks at one-loop order, assuming a small mass (< 10 MeV) for the gluon. Let us first describe what we plan to calculate, and the notation and conventions that we will use. The piece of the Lagrangian which governs the quark-gluon coupling for a non-zero value of the anomalous CMDM is given by

$$\mathcal{L}_{CMDM} = g_s \bar{\psi} T_a i F_2(q^2) \sigma_{\mu\nu} q^\nu \psi G^{\mu a}, \quad (1.1)$$

where ψ denotes the quark and $G^{\mu a}$ denotes the gluon, g_s is the strong coupling constant, T_a are the usual $SU(3)_c$ generators, and m is the quark mass. We will call $F_2(q^2)$ the CMDM, corresponding to a momentum transfer q . For future reference, we mention here that in [10, 33], which calculated the CMDM of the top quark, a quantity $\Delta\kappa$ was defined by

$$\frac{\Delta\kappa}{4m} = F_2(q^2 = 0) \quad (1.2)$$

and referred to as the CMDM of the quark.

The form factor $F_2(q^2)$ receives contributions from both strong and electroweak processes. The lowest order QCD contribution comes from two different Feynman diagrams, shown in

Fig. 1. For the diagram in Fig. 1(a) the external gluon directly couples to the fermion line in the loop, similar to the analogous process of quantum electrodynamics. The other diagram, shown in Fig. 1(b), is purely non-Abelian in nature, with the external gluon coupling to internal gluons.

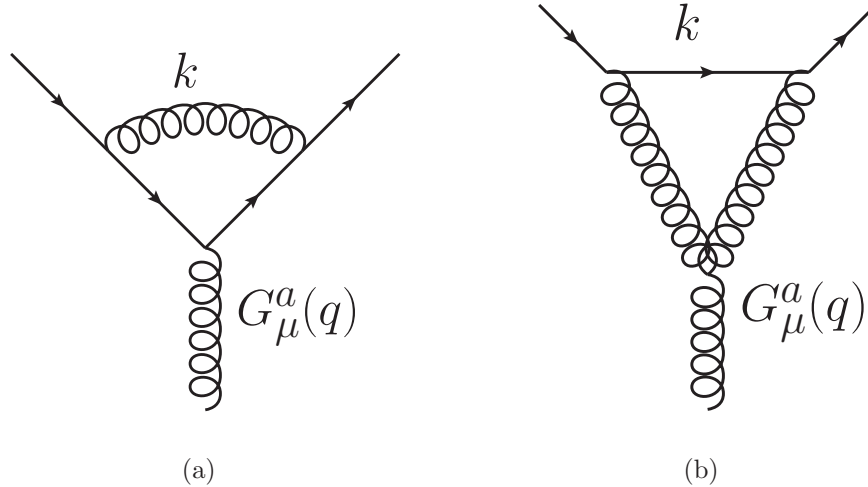


FIG. 1: Strong contributions to the anomalous chromomagnetic dipole moment of a quark: (a) QED-like diagram; (b) purely non-Abelian contribution.

The electroweak contribution to the CMDM comes from loops containing the exchange of electroweak gauge bosons γ, Z, W , or the Higgs boson. Using the observed value of the Higgs boson mass, $m_H = 125.9$ GeV [19] completely fixes the contribution from the electroweak sector. The relevant vertex corrections at the lowest order come from the diagrams shown in Fig. 2.

To begin with, we focus our attention on $F_2(0)$, calculating this quantity for all quarks and for varying gluon mass between 0 and 10 MeV. An analytical calculation of the anomalous CMDM was done in [10] and later in [33]. Also see [34] for a different method of calculation. Apart from the use of a massive gluon propagator, there is another significant way by which our calculations and results differ from those in these papers. The results and calculations in these papers were done for zero momentum exchange, i.e. $q^2 = 0$, whereas we do the calculations for a general q^2 . For $q^2 = 0$ the diagram of Fig. 1(b) produces a divergent contribution to the CMDM for vanishing gluon mass. While it is possible to get rid of this infrared divergence if we work with zero gluon mass from the beginning, this diagram cannot be subtracted out if the physical gluon has a mass. Then this diagram contributes

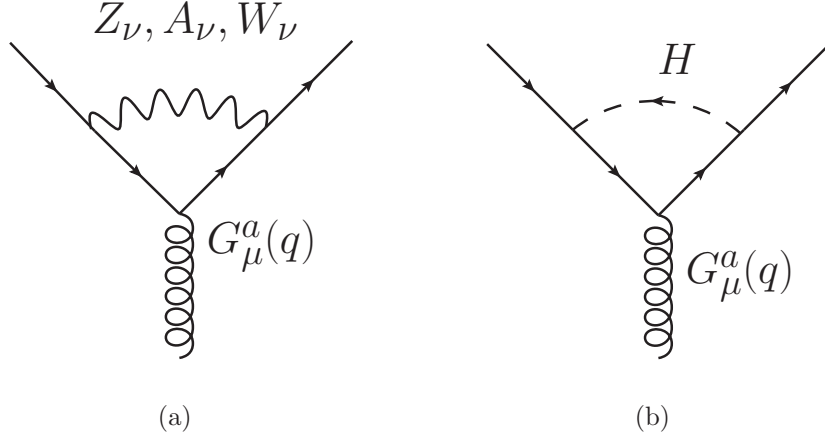


FIG. 2: Electroweak contributions to the anomalous chromomagnetic dipole moment of a quark: (a) gauge boson exchange; (b) Higgs boson exchange.

to the total, and $F_2(0)$ diverges as the gluon mass is taken to zero. We will argue below that perturbative $F_2(0)$ is not a physically sensible quantity anyway, and what we should be interested in is $F_2(q^2)$ for large values of $-q^2$.

We note here that while a calculation for the anomalous CMDM was given in [33], those calculations appear to be incorrect. Specifically, the numerical results given in [33] are in severe disagreement with the calculations given in the appendix of the same paper, one being finite and the other divergent. The results and calculations are both in disagreement with ours. Our results for F_2 at $q^2 = 0$ appear to agree with those in [34], whose result for Fig. 1(b) also diverges at $q^2 = 0$ but vanishes for vanishing quark mass, as does ours. Our formula for Z -exchange agrees with a similar formula in [35].

In Sec. II we provide an outline of the calculation of various contributions to the CMDM for $q^2 = 0$. We then discuss the shortcomings of these results and in Sec. III provide the results for $q^2 = -M_Z^2$.

II. CALCULATIONS

The calculations proceed in a straightforward manner. We display only the main steps for the calculation of the diagrams in Figs. 1 and 2. We will calculate the form factor F_2 at $q^2 = 0$, and we will take the gluon propagator to be

$$\Delta_{\mu\nu}^{ab} = -i\delta^{ab} \frac{g_{\mu\nu}}{k^2 - M^2 + i\epsilon}, \quad (2.1)$$

M being the mass of the gluon. There will also be a term proportional to $k^\mu k^\nu$ in the propagator, the actual form of the term depending on the theory that provides a mass to the gluon. But in the diagrams of Fig. 1, each internal gluon line couples to a conserved $U(1)$ current at least at one end. So this term will not contribute in these diagrams. The calculations below are relevant for of the dynamical mass generation model and the Curci-Ferrari model; the topological mass generation model has additional diagrams and will be considered elsewhere.

A. Strong contribution

The contribution of the diagram in Fig. 1(a) to the vertex function Γ_μ can be written as

$$\Gamma_\mu^{(1a)}(q)T_{ji}^a = -\frac{i}{6}g_s^2 T_{ji}^a \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\lambda(\not{p}' + \not{k} + m)\gamma_\mu(\not{p} + \not{k} + m)\gamma^\lambda}{[(p' + k)^2 - m^2][(p + k)^2 - m^2](k^2 - M^2)}, \quad (2.2)$$

where we have used $T_{jk}^b T_{ki'}^a T_{i'i}^b = -\frac{1}{6}T_{ji}^a$. A straightforward calculation gives us the coefficient of $i\sigma_{\mu\nu}q^\nu$, which is the contribution of Fig. 1(a) to the form factor $F_2(q^2)$,

$$F_2^{(1a)}(q^2) = -\frac{8i\alpha_s\pi m}{3} \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\ \times \frac{(\zeta_1 + \zeta_2)(1 - \zeta_1 - \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m^2 + \zeta_1 \zeta_2 q^2 - \zeta_3 M^2]^3}. \quad (2.3)$$

For $q^2 = 0$, we get

$$F_2^{(1a)}(0) = -\frac{\alpha_s}{12m\pi} \int_0^1 d\zeta \frac{(1 - \zeta)^2 \zeta}{(1 - \zeta)^2 + \zeta \lambda^2}, \quad (2.4)$$

where we have defined

$$\lambda = \frac{M}{m}. \quad (2.5)$$

The result of this integral is included in the plot of Fig. 3, using $\alpha_s(M_Z)$.

The value of $F_2(0)$ for the top quark was calculated in [10] for vanishing gluon mass, using only the diagram of Fig. 1(a). Our result agrees with that value. The calculation for this diagram, as given in Eq. (A5) of [33], gives a divergent result and is in disagreement with our calculations above.

We now calculate the other part of the strong contribution to the anomalous chromomagnetic dipole moment coming from Fig. 1(b). The one loop contribution to the vertex

function Γ_μ for this diagram is given by

$$\Gamma_\mu^{(1b)} = -i \frac{g_s^2}{4} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\lambda (\not{k} + m) \gamma^\nu}{(k^2 - m^2)[(k - p)^2 - M^2][(k - p')^2 - M^2]} \times [(-2k + p + p')_\mu g_{\nu\lambda} + (k + p - 2p')_\nu g_{\lambda\mu} + (k + p' - 2p)_\lambda g_{\mu\nu}], \quad (2.6)$$

where we have used

$$T_{ji'}^c T_{i'i}^b f_{abc} = -\frac{i}{4} T_{ji}^a. \quad (2.7)$$

Again after a bit of algebra, we get the contribution from Fig. 1(b) to the form factor $F_2(q^2)$,

$$F_2^{(1b)}(q^2) = i4\pi m\alpha_s \int \frac{d^4 k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \times \frac{(1 - \zeta_1 - \zeta_2)(\zeta_1 + \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m^2 - (\zeta_1 + \zeta_2)M^2 + (\zeta_1 + \zeta_2 - \zeta_3)m^2 + \zeta_1\zeta_2 q^2]^3}, \quad (2.8)$$

which for $q^2 = 0$ gives

$$F_2^{(1b)}(0) = \frac{\alpha_s}{8m\pi} \int_0^1 d\zeta \frac{\zeta(1 - \zeta)^2}{\zeta^2 + (1 - \zeta)\lambda^2}. \quad (2.9)$$

It was claimed in [10] that this contribution to the anomalous chromomagnetic dipole moment of the top quark vanished, for zero gluon mass. However, here we see that the integral diverges for $\lambda = 0$. We note that the formula (A12) in [33], corresponding to this diagram, also diverges. A divergence is also found in [34], vanishing for zero quark mass, as happens for Eq. (2.8).

We will come back to this point about the divergence later. Now we add the results of Eq. (2.4) and Eq. (2.9), and find the total contribution of strong interactions to the anomalous CMDM of a quark of mass m at one loop,

$$F_2^s(0) = -\frac{\alpha_s}{24m\pi} \int_0^1 d\zeta \frac{\zeta(1 - \zeta)(2 - 5\zeta)}{(1 - \zeta)^2 + \zeta\lambda^2}. \quad (2.10)$$

B. Electroweak contribution

Next we calculate the one loop contributions to the anomalous CMDM of a quark when the internal line is an electroweak gauge boson or a Higgs boson. The corresponding diagrams are given in Fig. 2.

Let us first consider Fig. 2(a), for the case where the internal line is a Z boson and the quark is an up-type quark. The contribution of this diagram to the vertex function γ_μ is

$$\Gamma_\mu^Z = \frac{2ie^2}{\cos^2 \theta \sin^2 \theta} \int \frac{d^4 k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \frac{N_\mu^Z(k) \delta(1 - \zeta_1 - \zeta_2 - \zeta_3)}{D_Z^3}, \quad (2.11)$$

where we have written

$$D_Z = k^2 + 2k(\zeta_1 p' + \zeta_2 p) - \zeta_3 M_Z^2, \quad (2.12)$$

and

$$N_\mu^Z(k) = \gamma_\nu \left(-\frac{1}{2}L + \frac{2}{3} \sin^2 \theta \right) (\not{k} + \not{p}' + m_u) \gamma_\mu (\not{k} + \not{p} + m_u) \gamma^\nu \left(-\frac{1}{2}L + \frac{2}{3} \sin^2 \theta \right). \quad (2.13)$$

Here m_u is the mass of the up-type quark, M_Z is the mass of the Z boson, and $\theta \equiv \theta_W$ is the weak mixing angle. As before, after a little algebra, we can rewrite the integral as

$$\begin{aligned} \Gamma_\mu^Z = & \frac{2ie^2}{(\cos \theta \sin \theta)^2} \int \frac{d^4 k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\ & \times \frac{N_\mu^Z(k - \zeta_1 p' - \zeta_2 p) + N_\mu^Z(k - \zeta_1 p - \zeta_2 p')}{2 [k^2 - (\zeta_1 + \zeta_2)^2 m^2 + \zeta_1 \zeta_2 q^2 - \zeta_3 M_Z^2]^3}. \end{aligned} \quad (2.14)$$

Ignoring all terms that do not contribute to $F_2(q^2)$, such as those proportional to γ_5 , we find that the contribution from $N_\mu^Z(k - \zeta_1 p' - \zeta_2 p)$ is

$$m \left[-\frac{1}{2}(1 - \zeta_1) + \left(-\frac{4}{3} \sin^2 \theta + \frac{16}{9} \sin^4 \theta \right) \zeta_1 \right] (1 - \zeta_1 - \zeta_2). \quad (2.15)$$

Similarly, the contribution from $N_\mu^Z(k - \zeta_2 p' - \zeta_1 p)$ is

$$m \left[-\frac{1}{2}(1 - \zeta_2) + \left(-\frac{4}{3} \sin^2 \theta + \frac{16}{9} \sin^4 \theta \right) \zeta_2 \right] (1 - \zeta_1 - \zeta_2). \quad (2.16)$$

Combining Eq.s (2.15), (2.16) and (2.14), we find that the contribution to the anomalous CMDM of up-type quarks from a Z -mediated diagram in Fig. 2(a) is given by

$$\begin{aligned} F_2^Z(q^2) = & -4\sqrt{2}iG_F M_Z^2 m_u \int \frac{d^4 k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\ & \times \frac{(\zeta_1 + \zeta_2 - 1) + \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta + \frac{16}{9} \sin^4 \theta \right) (\zeta_1 + \zeta_2)(1 - \zeta_1 - \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m_u^2 + \zeta_1 \zeta_2 q^2 - \zeta_3 M_Z^2]^3}, \end{aligned} \quad (2.17)$$

so that

$$F_2^Z(0) = -\frac{G_F M_Z^2}{4\sqrt{2}\pi^2 m_u} \int_0^1 d\zeta \frac{\zeta(1-\zeta) \left[-1 + \left(\frac{1}{2} - \frac{4}{3}\sin^2\theta + \frac{16}{9}\sin^4\theta\right)(1-\zeta)\right]}{(1-\zeta)^2 + \zeta\lambda_{Z,u}^2}, \quad (2.18)$$

where we have written $\lambda_{Z,u} = \frac{M_Z}{m_u}$. Our result Eq. (2.17) agrees exactly, after integration over k , with Eq. (A4) of [35] upon making the substitutions appropriate to an up-type quark,

$$a = \frac{2}{3} \frac{g'^2}{(g^2 + g'^2)^{\frac{1}{2}}}, \quad b = \frac{g'^2 - 3g^2}{(g^2 + g'^2)^{\frac{1}{2}}}, \quad (2.19)$$

where a and b are coupling constants used in [35].

Similarly, the contribution from a Z -mediated diagram in Fig 2(a) to the anomalous CMDM of a down type quark is

$$F_2^Z(0) = -\frac{G_F M_Z^2}{4\sqrt{2}\pi^2 m_d} \int_0^1 d\zeta \frac{\zeta(\zeta-1) + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta + \frac{4}{9}\sin^4\theta\right)\zeta(1-\zeta)^2}{(1-\zeta)^2 + \zeta\lambda_{Z,d}^2}, \quad (2.20)$$

with $\lambda_{Z,d} = \frac{M_Z}{m_d}$, where m_d is the mass of the down-type quark.

Next we consider the diagram Fig. 2(a) with a photon line in the loop. The calculation for this diagram is completely straightforward. For up-type quarks we calculate the contribution to be

$$F_2^A(0) = \frac{\alpha}{9\pi m_u}. \quad (2.21)$$

Similarly, the contribution of the photon-mediated diagram in Fig. 2(a) to the anomalous CMDM of a down-type quark is

$$F_2^A(0) = \frac{\alpha}{36\pi m_d}. \quad (2.22)$$

When a W is in the loop of Fig. 2(a), the contribution of the diagram to the vertex function of an up-type quark is

$$\Gamma_\mu^W = \frac{ie^2}{2\sin^2\theta} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\nu L(\not{k} + \not{p}' + m_d)\gamma_\mu(\not{k} + \not{p} + m_d)\gamma^\nu L}{[(k+p')^2 - m_d^2][(k+p)^2 - m_d^2](k^2 - M_W^2)}. \quad (2.23)$$

The form factor is easy to calculate,

$$F_2^W(0) = -\frac{G_F M_W^2}{4\sqrt{2}\pi^2 m_d} \int_0^1 d\zeta \frac{(1-\zeta^2)\zeta}{(1-\zeta)^2 + \zeta\lambda_{W,d}^2}, \quad (2.24)$$

where we have written, similarly to earlier definitions, $\lambda_{W,d}^2 = \frac{M_W^2}{m_d^2}$.

We similarly calculate the contribution of the W -mediated diagram to the anomalous CMDM of a down type quark to be

$$F_2^W(0) = -\frac{G_F M_W^2}{4\sqrt{2}\pi^2 m_u} \int_0^1 d\zeta \frac{(1-\zeta^2)\zeta}{(1-\zeta)^2 + \zeta\lambda_{W,u}^2}, \quad (2.25)$$

with $\lambda_{W,u}^2 = \frac{M_W^2}{m_u^2}$.

Finally, we consider the diagram with the Higgs boson in the loop of Fig. 2(a). The vertex function for a quark for this diagram is

$$\Gamma_\mu^H = -\frac{ie^2}{2\sin^2\theta} \frac{m^2}{M_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k} + \not{p}' + m)\gamma_\mu(\not{k} + \not{p} + m)}{((p+k)^2 - m^2)((p'+k)^2 - m^2)(k^2 - M_H^2)}. \quad (2.26)$$

Here M_H is the mass of the Higgs boson. Proceeding as in the previous cases, we obtain the contribution to the anomalous CMDM of both up-type and down-type quarks,

$$F_2^H(0) = \frac{G_F m}{4\sqrt{2}\pi^2} \int_0^1 d\zeta \frac{(1+\zeta)(1-\zeta)^2}{(1-\zeta)^2 + \zeta\lambda_{H,i}^2}. \quad (2.27)$$

with $\lambda_{H,i}^2 = \frac{M_H^2}{m_i^2}$ and i denotes the quark flavor.

In Table I we display the total contribution to $\Delta\kappa = 4mF_2(0)$ from weak interactions for each quark. The integrations were done using Mathematica [36]. For these calculations,

Quark	Z	A	W	H	Total
u	0	10.32×10^{-4}	0	0	10.32×10^{-4}
d	0	2.58×10^{-4}	0	0	2.58×10^{-4}
c	0	10.32×10^{-4}	0	0	10.32×10^{-4}
s	0	2.58×10^{-4}	0	0	2.58×10^{-4}
t	26.26×10^{-4}	10.32×10^{-4}	-3.93×10^{-4}	154.61×10^{-4}	187.26×10^{-4}
b	0	2.58×10^{-4}	-1.01×10^{-4}	0	1.57×10^{-4}

TABLE I: Weak contribution to the quark anomalous CMDM $\Delta\kappa$

we have taken $\alpha = (137.036)^{-1}$, the Fermi constant $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$, and the current quark masses $m_u = 2.3 \text{ MeV}$, $m_d = 4.8 \text{ MeV}$, $m_s = 95 \text{ MeV}$, $m_c = 1.275 \text{ GeV}$, $m_b = 4.18 \text{ GeV}$, as given in [19]. The zeroes in the table represent numbers smaller than 10^{-7} .

To these numbers we have to add the contributions from the strong interactions. The result of adding only Eq. (2.4) to the weak contributions is plotted as a function of gluon mass in Fig. 3. This number does not vary significantly with the mass of the gluons for the heavier quarks. That is expected, as the dependence on gluon mass M is through the ratio M/m , so for large enough m , small variations in M become negligible.

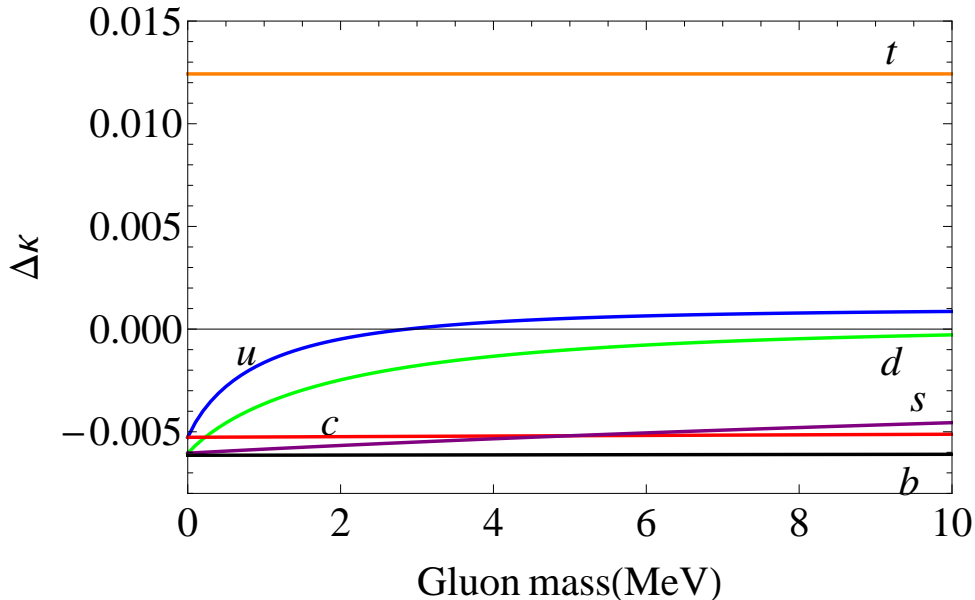


FIG. 3: Contribution to the anomalous CMDM from weak interactions and Fig. 1(a).

The contribution from Fig. 1(b) is infrared divergent when the gluon is massless. This is because the three-gluon vertex in Fig. 1(b), with all gluons massless, causes a divergence called the mass singularity [37, 38], when the external gluon is on-shell. If the gluon has a non-zero mass, this diagram contributes finitely to the anomalous CMDM of a quark. As mentioned earlier, if the gluon is taken to be massless from the beginning, it is possible to remove this divergence, but if the gluon has a mass, this diagram cannot be removed, and its contribution will diverge when the gluon mass is taken to zero.

We can see this in Fig. 4, where we have plotted the total $\Delta\kappa$ for each of the quarks, as a function of gluon mass. The contribution from Fig. 1(b) dominates, and the plots go to negative infinity as the gluon mass is taken to zero, but rise quickly as the gluon mass is increased.

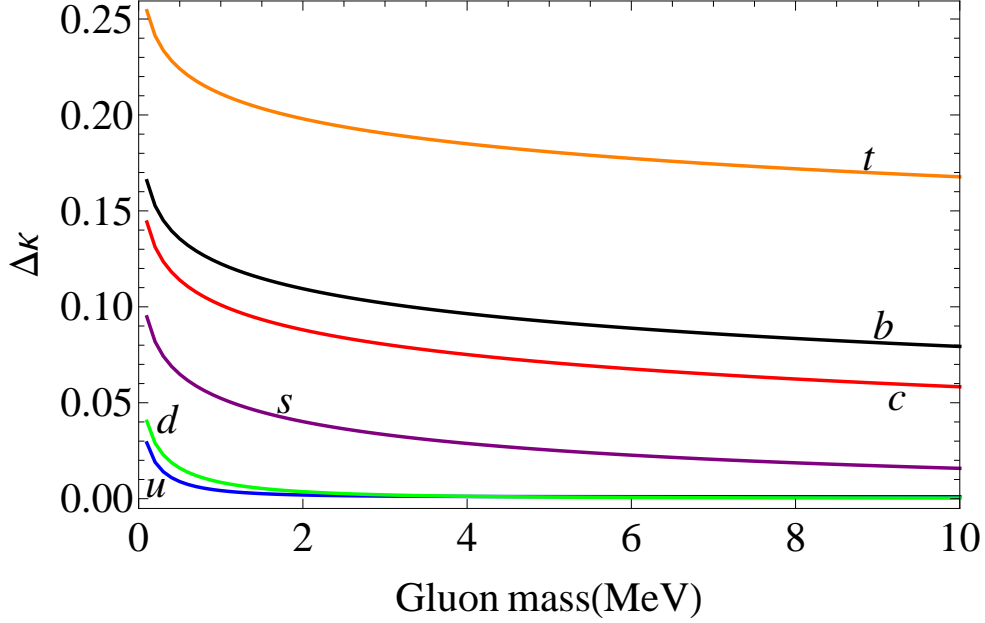


FIG. 4: Anomalous CMDM of quarks: dependence on gluon mass

III. ANOMALOUS CHROMOMAGNETIC MOMENT AT $q^2 = -M_Z^2$

In the previous section we calculated the anomalous chromomagnetic dipole moment of each quark by calculating the quark-gluon vertex form factor $F_2(q^2)$ at $q^2 = 0$. However, this definition is problematic, since it requires using perturbation theory at zero energy, where it is not valid for strong interactions. A related issue is that the measured values of the physical constants and masses pertaining to strong interactions are known at high energies, not at $q^2 = 0$. However, we can use the same techniques to calculate the form factor $F_2(q^2)$, and thus the anomalous CMDM at a higher energy scale. Let us calculate F_2 at energy corresponding to the Z -mass, i.e. at $q^2 = -M_Z^2$.

As before, we calculate F_2 by adding up the contributions from the diagrams in Figs 1

and 2. Using Eq. (2.3) we can write the contribution from the diagram in Fig. 1(a) as

$$\begin{aligned}
F_2^{(1a)}(q^2 = -M_Z^2) &= -\frac{8i\alpha_s\pi m}{3} \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\
&\quad \times \frac{(\zeta_1 + \zeta_2)(1 - \zeta_1 - \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m^2 - \zeta_1 \zeta_2 M_Z^2 - \zeta_3 M^2]^3} \\
&= -\frac{\alpha_s}{12\pi m} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{(1 - \zeta_3)\zeta_3}{(1 - \zeta_3)^2 + (1 - \zeta_2 - \zeta_3)\zeta_2 \lambda_{Z,i}^2 + \zeta_3 \lambda^2} \quad (3.1)
\end{aligned}$$

for the quark flavor i , with λ and $\lambda_{Z,i}$ as defined earlier. Similarly, the contribution from Fig. 1(b) is given by Eq. (2.8) at $q^2 = -M_Z^2$,

$$\begin{aligned}
F_2^{(1b)}(q^2 = -M_Z^2) &= 4i\pi m\alpha_s \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\
&\quad \times \frac{(1 - \zeta_1 - \zeta_2)(\zeta_1 + \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m^2 - (\zeta_1 + \zeta_2)M^2 + (\zeta_1 + \zeta_2 - \zeta_3)m^2 - \zeta_1 \zeta_2 M_Z^2]^3} \\
&= \frac{\alpha_s}{8\pi m} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{(1 - \zeta_3)\zeta_3}{\zeta_3^2 + (1 - \zeta_2 - \zeta_3)\zeta_2 \lambda_{Z,i}^2 + (1 - \zeta_3)\lambda^2}. \quad (3.2)
\end{aligned}$$

Next we consider Fig. 2(a) for an up-type quark with a Z boson in the internal line. Using Eq. (2.17) we calculate the contribution of this diagram to the form factor $F_2(-M_Z^2)$ as

$$\begin{aligned}
F_2^{(Z)}(q^2 = -M_Z^2) &= -\frac{im_u e^2}{\cos^2 \theta \sin^2 \theta} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \int \frac{d^4k}{(2\pi)^4} \delta(1 - \zeta_1 - \zeta_2 - \zeta_3) \\
&\quad \times \frac{(\zeta_1 + \zeta_2 - 1) + \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta + \frac{16}{9} \sin^4 \theta\right) (\zeta_1 + \zeta_2)(1 - \zeta_1 - \zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 m_u^2 - \zeta_1 \zeta_2 M_Z^2 - \zeta_3 M_Z^2]^3}, \\
&= -\frac{G_F M_Z^2}{4\sqrt{2}\pi^2 m_u} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{-\zeta_3 + \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta + \frac{16}{9} \sin^4 \theta\right) \zeta_3(1 - \zeta_3)}{(1 - \zeta_3)^2 + \lambda_{Z,u}\zeta_2(1 - \zeta_2 - \zeta_3) + \lambda_{Z,u}\zeta_3}. \quad (3.3)
\end{aligned}$$

Similarly, the contribution from Fig. 2(a) for a down type quark with an internal Z boson line is given by

$$F_2^{(Z)}(q^2 = -M_Z^2) = -\frac{G_F M_Z^2}{4\sqrt{2}\pi^2 m_d} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{-\zeta_3 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta + \frac{4}{9} \sin^4 \theta\right) \zeta_3(1 - \zeta_3)}{(1 - \zeta_3)^2 + \lambda_{Z,d}\zeta_2(1 - \zeta_2 - \zeta_3) + \lambda_{Z,d}\zeta_3}. \quad (3.4)$$

For the diagram in Fig. 2(a) with photon in the loop, the contribution for up-type quarks is

$$F_2^A(q^2 = -M_Z^2) = \frac{2\alpha}{9\pi m_u} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{\zeta_3(1-\zeta_3)}{(1-\zeta_3)^2 + \zeta_2(1-\zeta_2-\zeta_3)\lambda_{Z,u}^2}, \quad (3.5)$$

while for the down type quarks it is

$$F_2^A(q^2 = -M_Z^2) = \frac{\alpha}{18\pi m_d} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{\zeta_3(1-\zeta_3)}{(1-\zeta_3)^2 + \zeta_2(1-\zeta_2-\zeta_3)\lambda_{Z,d}^2}. \quad (3.6)$$

Next we consider the diagram in Fig. 2(a) with a W boson in the loop. According to Eq. (2.23) the contribution to $F_2(q^2 = -M_Z^2)$ for an up type quark is

$$\begin{aligned} F_2^W(q^2 = -M_Z^2) &= -\frac{ie^2 m_d}{\sin^2 \theta} \int_0^1 d\zeta_1 \int_0^1 d\zeta_2 \int_0^1 d\zeta_3 \int \frac{d^4 k}{(2\pi)^4} \delta(1-\zeta_1-\zeta_2-\zeta_3) \\ &\quad \times \frac{(1-\zeta_1-\zeta_2)(2-\zeta_1-\zeta_2)}{[k^2 - (\zeta_1 + \zeta_2)^2 - \zeta_1\zeta_2\lambda_{Z,d}^2 - \zeta_3\lambda_{W,d}^2]^3}, \\ &= -\frac{G_F M_W^2}{4\sqrt{2}\pi^2 m_d} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{\zeta_3(1-\zeta_3)}{(1-\zeta_3)^2 + \zeta_2(1-\zeta_2-\zeta_3)\lambda_{Z,d}^2 + \zeta_3\lambda_{W,d}^2}. \end{aligned} \quad (3.7)$$

Similarly, for the down type quark we obtain

$$F_2^W(q^2 = -M_Z^2) = -\frac{G_F M_W^2}{4\sqrt{2}\pi^2 m_u} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{\zeta_3(1-\zeta_3)}{(1-\zeta_3)^2 + \zeta_2(1-\zeta_2-\zeta_3)\lambda_{Z,u}^2 + \zeta_3\lambda_{W,u}^2}. \quad (3.8)$$

The contribution from the diagram in Fig. 2(b) to $F_2(q^2 = -M_Z^2)$ of a quark is obtained from Eq. (2.26) as

$$F_2^H(q^2 = -M_Z^2) = \frac{G_F m_i}{4\sqrt{2}\pi^2} \int_0^1 d\zeta_3 \int_0^{1-\zeta_3} d\zeta_2 \frac{(1+\zeta_3)(1-\zeta_3)}{(1-\zeta_3)^2 + \zeta_2(1-\zeta_2-\zeta_3)\lambda_{Z,i}^2 + \zeta_3\lambda_{H,i}^2}, \quad (3.9)$$

where i denotes the quark flavor as before.

In Table II we have collected the electroweak contributions to the form factor for the different quarks at $q^2 = -M_Z^2$. The values of G_F , m_i , and α were taken from [19]. The zeroes in this table represent numbers which are smaller by at least a factor of 10^{-3} than the smallest number in the same row. For ease of comparison with the zero momentum case, we

Quark	Z	A	W	H	Total
u	1.29×10^{-12}	29.79×10^{-12}	-4.41×10^{-12}	0	26.67×10^{-12}
d	5.50×10^{-12}	30.18×10^{-12}	-4.41×10^{-12}	0	31.27×10^{-12}
c	3.98×10^{-7}	36.91×10^{-7}	-0.48×10^{-7}	0	40.41×10^{-7}
s	0.22×10^{-8}	0.82×10^{-8}	-4.82×10^{-8}	0	-3.78×10^{-8}
t	25.66×10^{-4}	10.57×10^{-4}	-2.87×10^{-4}	149.91×10^{-4}	183.27×10^{-4}
b	4.16×10^{-6}	7.14×10^{-6}	-98.72×10^{-6}	0.03×10^{-6}	-87.39×10^{-6}

TABLE II: Electroweak contribution to $4mF_2(q^2 = -M_Z^2)$ of each quark

have given the values of the dimensionless quantity $4mF_2$. We note that unlike for $q^2 = 0$, this quantity differs by orders of magnitude between different quarks, clearly depending on the mass of the quark. This difference exists for the strong contributions as well, as we can see by adding to this the output of Eq.s (3.1) and (3.2). We have plotted the total for each quark in Fig. 5; clearly it is not meaningful to plot all of them in the same graph.

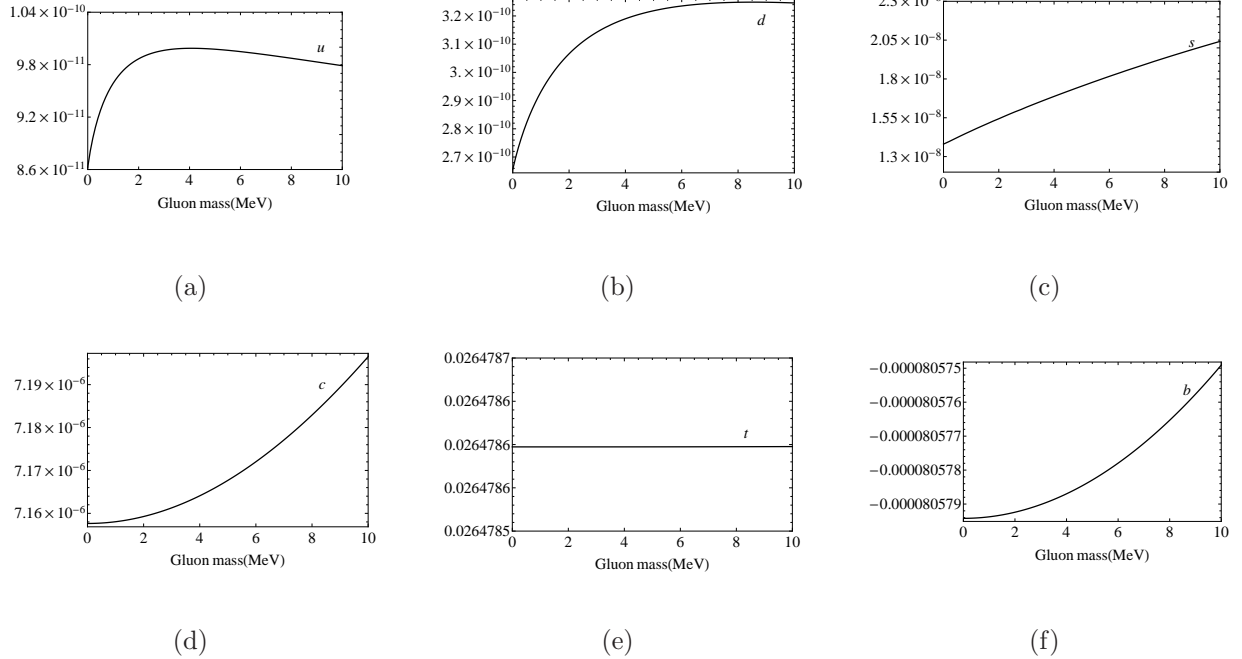


FIG. 5: Anomalous CMDM of quarks at M_Z : dependence on gluon mass

IV. DISCUSSION OF RESULTS

The anomalous chromomagnetic dipole moment of quarks may be defined analogously to the anomalous magnetic dipole moment. However, as we have seen in this paper, this analogy is not perfect. The non-Abelian nature of Yang-Mills theory leads to an additional diagram which diverges for vanishing gluon mass. If the gluon has a small mass, the divergence goes away; it is possible to set limits on the mass by using experimentally observed limits on $\Delta\kappa$.

In [10], $\Delta\kappa$ for the top quark was defined and calculated at zero momentum exchange (on-shell gluons), and there the bounds obtained were $|\Delta\kappa| \leq 0.45$ from Tevatron experiments, and a more stringent bound of $-0.03 \leq \Delta\kappa \leq 0.01$ from $b \rightarrow s\gamma$ transitions measured by the CLEO collaboration. But $\Delta\kappa$ is infrared divergent if the gluons are massless and that the more stringent bound is not satisfied by $\Delta\kappa$ for any gluon mass up to 10 MeV. The less stringent bound appears to be satisfied as long as the gluon mass is more than $\mathcal{O}(0.1 \text{ MeV})$. However, it is inappropriate to use perturbation theory to calculate the anomalous moments at $q^2 = 0$, and thus the relevant quantities should be calculated at some other value of q^2 .

We have therefore calculated the relevant form factor at the Z -mass, i.e. at $q^2 = -M_Z^2$. The results are now very different; there is no divergence for vanishing gluon mass. The dependence on gluon mass is most pronounced for the light quarks u, d and s , varying by 10 to 15% over a range of 0-10 MeV for the gluon mass.

Discussions about the anomalous chromomagnetic dipole moment in the literature have focused only on the top quark because it is larger in magnitude than for the other quarks. We have corrected the existing results for the top quark at $q^2 = 0$ and also given results at the Z mass. It is also seen from our results that the anomalous chromomagnetic dipole moments of the other heavy quarks at may have measurable values at $q^2 = -M_Z^2$ as well. Future experiments at the LHC may be able to impose more precise bounds on the anomalous chromomagnetic dipole moments of different quarks, thus putting more stringent bounds on the mass of gluons.

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